Self similarity in the Thurston's teapots

Chenxi Wu

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- Collaborators: Harry Bray, Diana Davis, Kathryn Lindsey, Giulio Tiozzo
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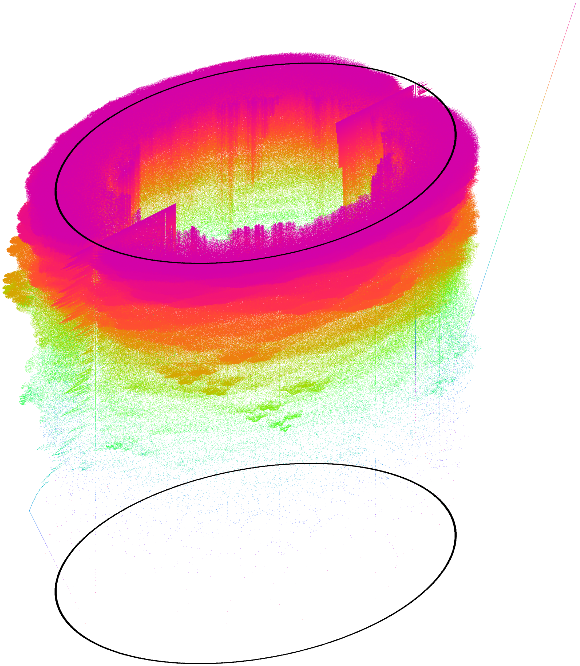
Thurston's Teapot

$$T = \overline{\{(z, \lambda) : \lambda = e^{h_f}, f \text{ unimodal map with periodic critical orbit },}$$
$$\overline{h_f \text{ topological entropy of } f, z \text{ Galois conjugate of } \lambda\}}$$

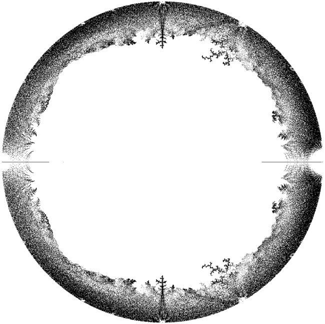
- Criteria for checking if a number can be such a topological entropy.
- Description (algorithm for verifying a point is not on the teapot): Bray-Davis-Lindsey-W
- Generalization: Lindsey-Tiozzo-W



Horizontal slices

Let $\lambda \in (1, 2)$, consider $\Omega_\lambda = \{z \in \mathbb{C} : |z| < 1, (z, \lambda) \in T\}$

The Ω_λ for $\lambda = (\sqrt{5} + 1)/2$:



Results

- Theorem 1: Given any $\lambda = e^{h_f}$ where f is a post critically finite unimodal map and h_f its topological entropy, there is a graph directed iterated function system parametrized by z, such that Ω_λ consists of those z that makes the limit set containing 1.
- Theorem 2: There are many points on Ω_λ with local self similarity. The proof is analogous to Tan Lei's proof of the Julia-Mandelbrot correspondence, and also similar to the works on IFS Mandelbrot set by Solomyak.
  Tan Lei's Four Conditions:
  - Bundle over parameter space is closed
  - Dense set of continuous sections
  - Self similarity of the fiber around 0
  - Smoothness of the number we want the "Julia set" to contain

Further Questions

- Calculation of the Hausdorff dimension
- How dense are the sets satisfying Theorem 2?