Dimensions of continued fraction Cantor sets

and their fractal cousins from the Fatou–Sullivan dictionary

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There are embedded hyperlinks in these slides, mostly in this color
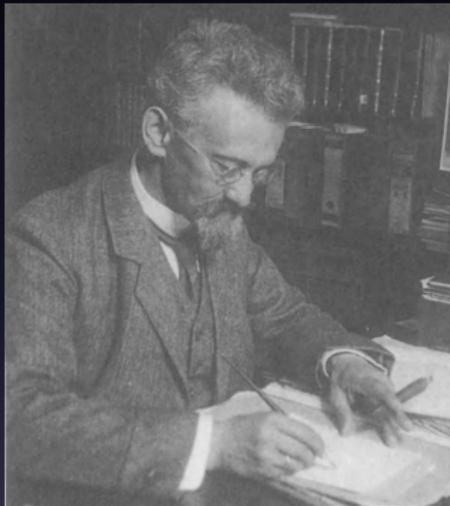
My brilliant collaborators, friends, teachers

Lior Fishman (UNT), Feliks Pryzytycki (IMPAN), David Simmons (York), Bernd Stratmann[†] (Bremen), Giulio Tiozzo (Toronto/Sapienza), Mariusz Urbański (UNT), Anna Zdunik (IMPAN)

Measuring fractal sets

Ich bin kein Naturforscher, sondern ein Mensch, der im Unendlichen seine Heimat sucht.
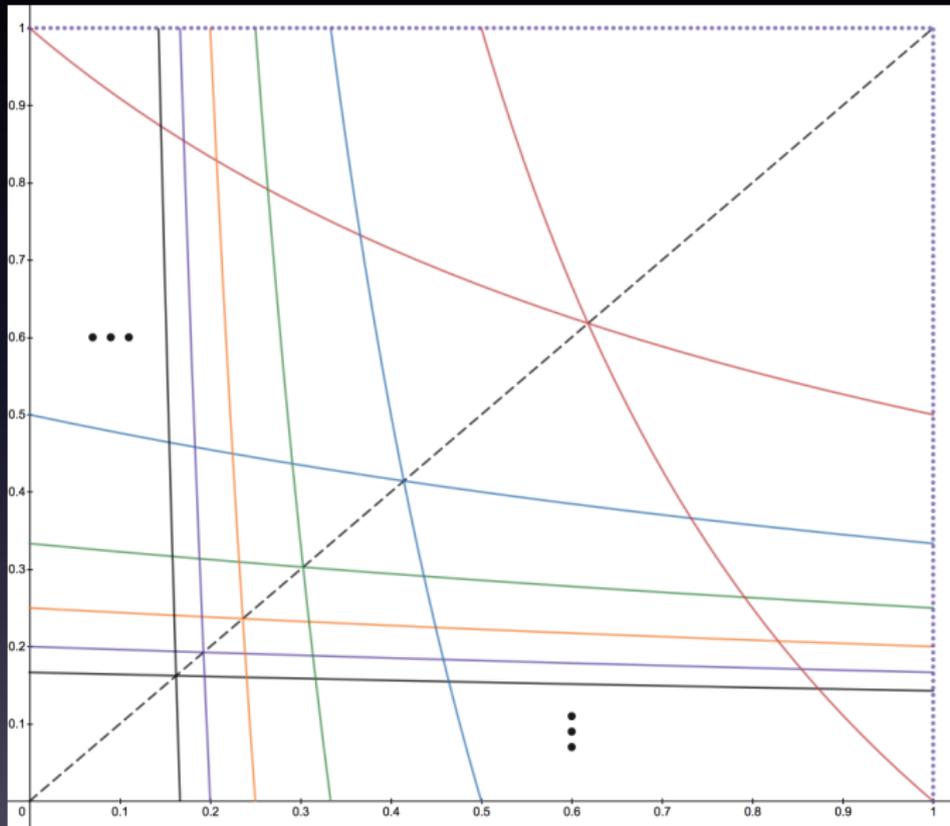

Felix Hausdorff / Paul Mongré
(November 8, 1868 – January 26, 1942)

Hausdorff dimension/measure allows studying intricate sets that are invisible to Lebesgue measure.

A way of measuring the "size" of such sets, such that the HD = TD of a smooth manifold.

A favorite CIFS via $x \mapsto 1/x \mod 1$

For each $i \in \mathbb{N}$, consider the map $g_i : [0, 1] \to [0, 1]$ defined by

$$g_i(x) = \frac{1}{i + x}$$

Each g_i is an inverse branch of the Gauss map $G : [0, 1] \to [0, 1]$, i.e.

$$G(x) = \begin{cases} 1/x - \lfloor 1/x \rfloor & x > 0 \\ 0 & x = 0 \end{cases}$$

where $\lfloor x \rfloor$ is the integer part of x.

For any set $E \subseteq \mathbb{N}$, called an alphabet, the collection of maps $\{g_i\}_{i \in E}$ is a Gauss CIFS. When the alphabet $E = \mathbb{N}$, $\{g_i\}_{i \in \mathbb{N}}$ is *the* Gauss CIFS.

Given $\omega = \omega_0\omega_1\omega_2\ldots\omega_{n-1} \in \mathbb{N}^n$, let

$$g_\omega := g_{\omega_0} \circ g_{\omega_1} \ldots \circ g_{\omega_{n-1}} : [0,1] \to [0,1]$$

so that

$$g_\omega(x) = \cfrac{1}{\omega_0 + \cfrac{1}{\omega_1 + \cfrac{1}{\ddots + \cfrac{1}{\omega_{n-1} + x}}}}$$

In particular,
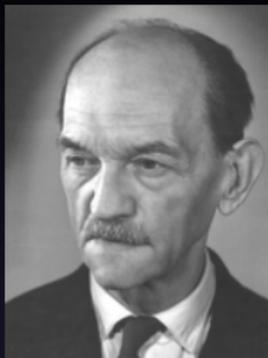
$$g_\omega(0) = [0; \omega_0, \omega_1, \ldots, \omega_{n-1}]$$

For any alphabet $E \subseteq \mathbb{N}$, the limit set of a Gauss CIFS $\{g_i\}_{i \in E}$ is

$$F_E := \bigcap_{n \in \mathbb{N}} \bigcup_{\omega \in I^n} g_\omega([0,1])$$

F_E is the set of irrationals in $[0,1]$ whose continued fraction digits lie in E.
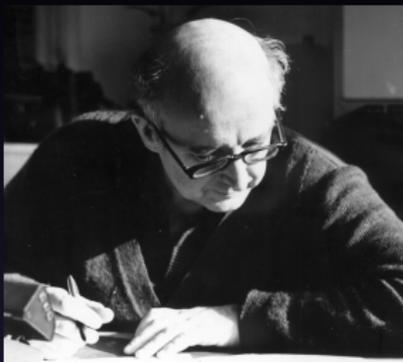
Jarník

Vojtěch Jarník

Let $F_{\leq N} := F_{\{1,2,\ldots,N\}}$ denote the set of irrationals in $[0, 1]$ whose continued fraction digits are all $\leq N$.

In 1928, Jarník established that for every $N \geq 8$

$$1 - \frac{4}{N\log(2)} \leq \dim_H(F_{\leq N}) \leq 1 - \frac{1}{8N\log(N)}$$

where $\dim_H$ denotes Hausdorff dimension

Kurzweil

Jaroslav Kurzweil

Kurzweil's thesis (1949) contains the following strengthening of Jarník's bounds, published in 1951:

$$1 - \frac{0.99}{N} \le \dim_H(F_{\le N}) \le 1 - \frac{0.25}{N}$$

Generalization to matrices (a.k.a. systems of linear forms) by Simmons (Math. Proc. Cambridge Phil. Soc, 2018)

Hensley

Doug Hensley

In 1992 Hensley improves on Kurzweil by proving

$$\dim_H(F_{\leq N}) = 1 - \frac{6}{\pi^2} \cdot \frac{1}{N} - \frac{72}{\pi^4} \cdot \frac{\log(N)}{N^2} + O\left(\frac{1}{N^2}\right)$$

…and for now, here ends this $F_{\leq N}$ tale. Let's return to the late 1930s …

Good

A. S. Besicovitch   I. J. Good   G. H. Hardy

"My graduate advisors (c.1937–1941) were Besicovitch and Hardy. I think what I most liked then was amazing formulas such as those of Ramanujan.

. . . (Besicovitch) suggested that I might investigate the fractional dimensions of sets of simple continued fractions defined in a simple manner. My research in this area was awarded a Smith Prize (1940). I found, for example, that the set for which the partial quotients a_n tend to infinity has fractional-dimensional number, or fractal number, $\frac{1}{2}$ (the fractal number of a set can be regarded as a measure of its texture, especially in two or three dimensions).

When $\sqrt[n]{a_n}$ is unbounded, the fractal number is again $\frac{1}{2}$, and if $a_n = 1$ or 2 for all n, the fractal number is about 0.53.

Good

Let $F_{\geq N} := F_{\{N, N+1, \dots\}}$ denote the irrationals in $[0, 1]$ whose continued fraction digits are all $\geq N$.
Good (1941) proved that for $N \geq 20$

$$\frac{1}{2} + \frac{1}{2\log(N+2)} \leq \dim_H(F_{\geq N}) \leq \frac{1}{2} + \frac{\log\log(N-1)}{2\log(N-1)}$$

Improving on Ramharter (1983) and Cusick (1977), Jaerisch–Kesseböhmer (2010) proved $\dim_H(F_{\geq N}) - 1/2 \sim \log\log(N)/2\log(N)$

Jarník et al redux

Theorem (DFSU, 2020)

For each $p \geq 1$, $\dim_H(F_{\leq N})$ can be estimated via the formula

$$\dim_H(F_{\leq N}) = 1 + \sum_{i=1}^{p-1} \sum_{j=0}^{i-1} c_{i,j} \frac{\log^j(N)}{N^i} + O_p\left(\frac{\log^{p-1}(N)}{N^p}\right)$$

where $c_{i,j} \in \mathbb{R}$ are computable constants. Here O_p means that the implied constant of O may depend on p.

Our methods yield explicit (but very complicated formulas) for subsequent $c_{i,j}$ coefficients

However, the following sequence of coefficients turns out to have a relatively simple expression:

$$c_{i,i-1} = -\frac{2^{i-1} \cdot i^{i-2}}{(i-1)!} \left(\frac{6}{\pi^2}\right)^i$$

which includes the two coefficients $c_{1,0}$ and $c_{2,1}$ computed by Hensley.

Good et al redux

Die Baukunst ist eine erstarrte Musik – Goethe

Theorem (DFSU, 2020)

For each $p \geq 1$, $\dim_H(F_{\geq N})$ can be estimated via the formula

$$
\dim_H(F_{\geq N}) = \frac{1}{2} + \frac{1}{2\log(N)} \left[\log\log(N) - \log\log\log(N) \right.
$$

$$
+ \sum_{k=1}^{\infty} \sum_{\ell=1}^{k} c_{k,\ell} \frac{\log^{\ell} \log\log(N)}{\log^k \log(N)}
$$

$$
+ \sum_{i=1}^{p-1} \sum_{j=1}^{\infty} \sum_{k=-j}^{\infty} \sum_{\ell=0}^{j+k} c_{i,j,k,\ell} \frac{\log^{\ell} \log\log(N)}{N^i \log^j(N) \log^k \log(N)} \left. \right]
$$

$$
+ O_p\left(\frac{\log\log(N)}{N^p \log(N)} \right)
$$

where $c_{k,\ell} \in \mathbb{Q}$ and $c_{i,j,k,\ell} \in \mathbb{Q}$ are computable.

A perturbation theorem (DFSU, 2020)

set up and notation:

$f \in \mathcal{B}$ a Banach space

$\sigma \in \mathcal{B}^* = \{ \sigma : \mathcal{B} \to \mathbb{C} \mid \sigma \text{ bdd linear} \}$

$L \in \mathcal{L}(\mathcal{B}) = \{ L : \mathcal{B} \to \mathcal{B} \mid L \text{ bdd linear} \}$

$$
\begin{array}{ccc}
\mathcal{B} & \mathcal{B}^* & \mathcal{L}(\mathcal{B}) \\
\cup & \cup & \cup \\
f & \sigma & L \\
\text{column} & \text{row} & \text{square}
\end{array}
$$

$\sigma f \in \mathbb{C}$

$f\sigma \in \mathcal{L}(\mathcal{B})$ $(f\sigma)f' = (\sigma f')f$

$\sigma L \in \mathcal{B}^*$ $(\sigma L)f = \sigma(Lf)$ ← if you prefer $\left(\sigma \xmapsto{L^*} \sigma L \right.$, $\left. (L^* \sigma)f = \sigma(Lf) \right)$

f is a right fixed pt of L if $Lf = f$

σ is a left fixed pt of L if $\sigma L = \sigma$

Theorem (DFSU, 2020) Sp. L, L' in $\mathcal{L}(\mathcal{B})$ have right fixed points $g, g' \in \mathcal{B}$, and $\mu \in \mathcal{B}^*$ be a left fixed point of L s.t. $\mu g, \mu g' \neq 0$.

Let

$$R \stackrel{\text{def}}{=} L - cg\mu \in \mathcal{L}(\mathcal{B})$$

where $c = 1/\mu g$, be such that $\rho(R) < 1$.

Let

$$Q \stackrel{\text{def}}{=} \sum_{n=0}^{\infty} R^n \in \mathcal{L}(\mathcal{B}) \quad \text{and} \quad \Delta \stackrel{\text{def}}{=} L' - L \in \mathcal{L}(\mathcal{B})$$

Then

$$\boxed{\sum_{n=0}^{\infty} \|R^n\| \cdot \|\Delta\| < 1 \quad \Rightarrow \quad \sum_{p=0}^{\infty} \mu\Delta(Q\Delta)^p g = 0}$$

$\star$ Can replace hypothesis $L'g' = g'$ s.t. $\mu g' \neq 0$ with $\mu'L' = \mu'$ s.t. $\mu'g \neq 0$

Consider transfer operators $L, L_{\leq N}, L_{\geq N} : C([0, 1]) \to C([0, 1])$

$$Lf(x) \stackrel{\text{def}}{=} \sum_{n=1}^{\infty} \frac{1}{(n+x)^2} f\left(\frac{1}{n+x}\right)$$

$$L_{\leq N}f(x) \stackrel{\text{def}}{=} \sum_{n=1}^{N} \frac{1}{(n+x)^{2\delta_{\leq N}}} f\left(\frac{1}{n+x}\right)$$

$$L_{\geq N}f(x) \stackrel{\text{def}}{=} \sum_{n=N}^{\infty} \frac{1}{(n+x)^{2\delta_{\geq N}}} f\left(\frac{1}{n+x}\right)$$

where $\delta_{\leq N}, \delta_{\geq N}$ are Hausdorff dimensions of $F_{\leq N}, F_{\geq N}$

The celebrity Gauss–Kuzmin–Wirsing operator L fixes the positive function $g(x) = 1/(1+x)$, and its dual L^* fixes μ, the Lebesgue measure on $[0, 1]$.

Now, $\delta_{\square N} = \dim_H(F_{\square N}) \Leftrightarrow \rho(L_{\square N}) = 1$, which is equivalent to $L_{\square N}$ fixing a positive function $g_{\square N}$ and the dual $(L_{\square N})^*$ fixing a positive measure $\mu_{\square N}$.

Proof strategy for $F_{\leq N}$

Apply perturbation theorem with L being GKW and $L' = L_{\leq N}$

As before, solve the resulting formula $\sum_{p=0}^{\infty} \mu \Delta (Q\Delta)^p g = 0$ for $\delta_{\leq N}$

Useful aids: Euler–Maclaurin formula & Implicit Function Theorem

$$\delta_{\leq N} = \dim_H(F_{\leq N}) = 1 + \sum_{i=1}^{p-1} \sum_{j=0}^{i-1} c_{i,j} \frac{\log^j(N)}{N^i} + O_p\left(\frac{\log^{p-1}(N)}{N^p}\right)$$

Proof strategy for $F_{\geq N}$

Now let $L' = L_{\geq N}$, except instead of taking L to be GKW let $L = h\nu$ where ν is the Dirac point mass at 0 and $h = \mathbb{1}$ (i.e. $Lf(x) = f(0)$).

Solve $\sum_{p=0}^{\infty} \mu\Delta(Q\Delta)^p g = 0$ for $\delta_{\geq N}$

$$
\begin{aligned}
\delta_{\geq N} = \dim_H(F_{\geq N}) = \frac{1}{2} + \frac{1}{2\log(N)} & \left[\log\log(N) - \log\log\log(N) \right. \\
& + \sum_{k=1}^{\infty} \sum_{\ell=1}^{k} c_{k,\ell} \frac{\log^{\ell}\log\log(N)}{\log^k\log(N)} \\
& + \sum_{i=1}^{p-1} \sum_{j=1}^{\infty} \sum_{k=-j}^{\infty} \sum_{\ell=0}^{j+k} c_{i,j,k,\ell} \frac{\log^{\ell}\log\log(N)}{N^i \log^j(N) \log^k\log(N)} \left.\right] \\
& + O_p\left(\frac{\log\log(N)}{N^p \log(N)} \right)
\end{aligned}
$$

what else can we do?

Our methods also give ` series expansions ` for several systems!

Let δ denote the HD of irrationals with Virahaṅka (a.k.a. Fibonacci) continued fraction digits, i.e. the alphabet is $1, 2, 3, 5, 8, 13, \ldots$
Now let $\delta_{\leq N}$ denotes the HD of irrationals with c.f. digits among the first N Virahaṅka numbers. Then

$$\delta_{\leq N} = \delta - \sum_{k=1}^{\infty} \sum_{i=0}^{\infty} \sum_{h=0}^{\infty} \sum_{j=0}^{k-1} c_{k,i,h,j} \left(\phi^{-(2\delta k + i)} \rho^h \right)^N N^j$$

where $\rho = \phi^*/\phi$, and ϕ, ϕ^* are the golden mean and its conjugate.

"What are generally referred to as the Fibonacci numbers and the method for their formation were given by **Virahaṅka** *(between AD 600 and 800),* **Gopala** *(prior to AD 1135) and* **Hemachandra** *(c. AD 1150), all prior to L. Fibonacci (c. AD 1202)."* – P. Singh, The so-called Fibonacci numbers in ancient and medieval India, Historia Math. (1985) **12** (3) 229–244.

We have an analogous (convergent series) formula for

$$\delta_{\geq N} = \frac{1}{2\log(\phi)} \frac{1}{N} \left[\log(N) - \log\log(N) + \right.$$

$$\left. + \sum_{i=0}^{\infty} \sum_{h=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=-j}^{\infty} \sum_{\ell=0}^{j+k} c_{i,h,j,k,\ell} (\phi^{-i}\rho^h)^N \frac{\log^\ell \log(N)}{N^j \log^k(N)} \right]$$

Coefficients in the $\delta_{<N}$ series may be computed to arbitrary precision using integrals, while coefficients in the $\delta_{\geq N}$ series are in the ring

$$\mathbb{Q}(\phi, \rho) \left[\frac{1}{\sqrt{5}}, \log\left(\frac{1}{\sqrt{5}}\right), \log(\phi), \log(\rho) \right]$$

and may be computed numerically to arbitrary precision.

Questions, puzzles, directions &c.

- 2 sample real CIFSs where our tech doesn't directly apply:
  - ▸ Prime continued fraction Cantor set $x \mapsto (p + x)^{-1}$ for primes p
  - ▸ co-Cantor infinite Similarity IFS (generators map gaps to $[0, 1]$)

- Study alphabets given by constant-recursive, holonomic, automatic . . . sequences

- Find analogues of our results for complex continued fractions!

  Chousionis–Leykekhman–Urbański, (Selecta, 2019)

- Try to work out an infinite-alphabet self-affine IFS example!

  Jurga (Selecta, 2021), Reeve (Israel J., 2010)

- Investigate analogs for holomorphic/meromorphic Julia sets and Kleinian group limit sets. Bodart–Zinsmeister (1996), Soares (2020), Li–Matheus–Pan–Tao (2024), Dang–Mehmeti (2024), Guilloux–Courtois (2024), Talbott (2025)

- Leverage our perturbation theorem to analyze open systems and conformal GDMS E.g., Edgeworth expansions, derivatives of pressure &c.
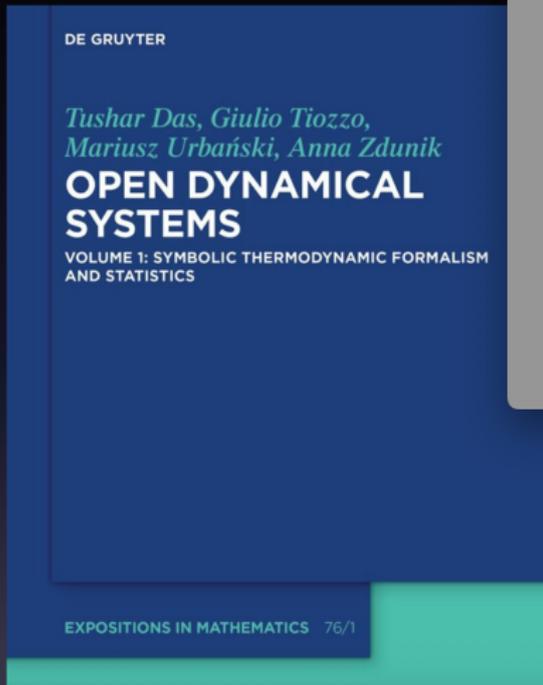
Connection $\rightsquigarrow$ $F_{\leq N}$ is the (survivor) set of points that do not fall into the hole $[0, 1/N + 1)$ under forward iteration of the Gauss map.

Fatou–Sullivan (bilingual) dictionary

Sullivan, Seminar on conformal and hyperbolic geometry (IHÉS,1982)
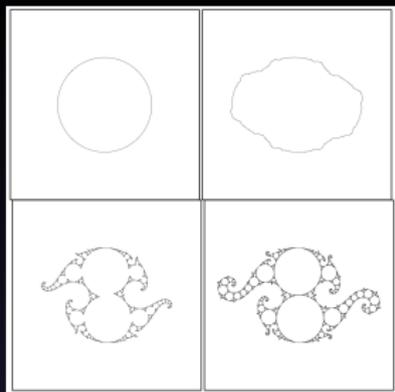 –, Quasiconformal homeos and dynamics I (Annals, 1985)
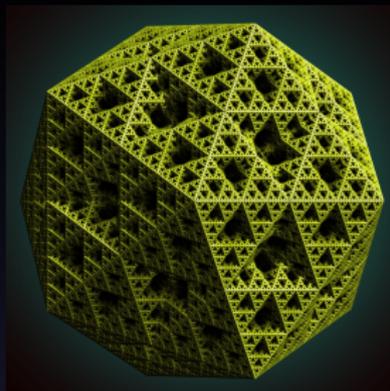 –, Quasiconformal homeos and dynamics II (Acta, 1985)

Curt McMullen

Limit sets, Curt McMullen (1976), Dennis Sullivan (c. 1980), Julia sets

Drawings of Limit sets & Julia sets by Curt McMullen
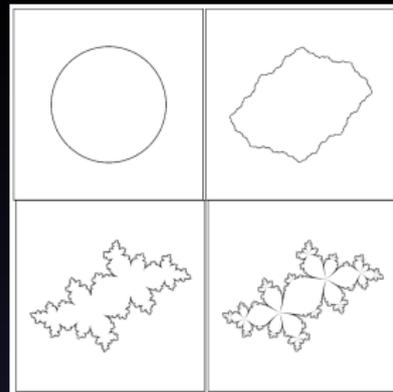
…Fatou–Sullivan's dictionary is akin to a big–bang event
with several directions in which to investigate extensions …

Limit sets   IFS attractors   Julia sets

Drawings of Limit sets & Julia sets by Curt McMullen, IFS drawing by Jos Leys.

- **coarse/metric geometry and dynamics**
  Bonk–Meyer, Haissinsky–Pilgrim, Li, D.–Przytycki–Tiozzo –Urbański–Zdunik …
- **non-archimedean geometry and dynamics**
  DeMarco, Manin–Marcolli, Rivera-Letelier, Li–Matheus–Pan–Tao …
- **stochastic/probabilistic geometry**
  Lin–Rohde, Bonk–Tran … Curien, Le Gall, Miller, Sheffield, Rhodes, Vargas …
- **Matings, reflections, correspondences**
  Bullett–Lomonaco, Lyubich–Mukherjee …
- **Anosov groups**
  Kapovich–Leeb–Porti, Bridgeman–Canary–Labourie–Sambarino, Kassel, Weinhard, Oh, …

A Ψ-lingual dictionary

DSU (Advances, 2017)

Find analogues of our results in Kleinian/GDMS/Julia/…realms!

When studying a concept/theorem in any column ask what the other analogous entries might be ☺

Kleinian groups and their generalizations	Conformal IFSes and GDMSes	Rational and meromorphic functions
Limit set	Closure of limit set	Julia set
Radial limit set	Limit set	Radial Julia set
Discreteness assumption	Open set condition	??
Poincaré exponent	Unique zero of pressure	Poincaré exponent Unique zero of pressure Hyperbolic dimension Dynamical dimension
Bishop–Jones theorem	Bowen's formula	Bowen-type formula
Convex-cobounded group	Finite IFS/GDMS	NCP map with no parabolic points e.g. hyperbolic rational maps
Geometrically finite group	Finite parabolic IFS/GDMS	NCP map e.g. parabolic & geometrically finite rational maps
Patterson–Sullivan measure	δ-conformal measure	δ-conformal measure
Patterson density of a Gibbs cocycle	Equilibrium/Gibbs state of a potential function	Equilibrium/Gibbs state of a potential function

A decade-old example of such Ψ-research

Around 2014, while trying to make sense of

Theorem (Kapovich, GAFA, 2009)

Let $G \leq \text{Mob}(\mathbb{R}^d)$ be a nonelementary geometrically finite Kleinian group with limit set Λ. Then
   either $\dim_H(\Lambda) > k := \dim_T(\Lambda)$ or Λ is a generalized k-sphere.

A *generalized k-sphere* in $\mathbb{R}^d$ is either a k-dimensional plane or sphere contained in a $(k + 1)$-dimensional affine subspace of $\mathbb{R}^d$.

We wanted to generalize and unify previous rigidity results in the three classical conformal realms, and also extend such results to

infinite dimensions for Kleinian groups and GDMS.

Theorem (D.–Simmons–Urbański, Advances, 2017)

Let Λ be the limit set of a conformal dynamical system, i.e. a Kleinian group, a CIFS, or a rational map.
If $\mathcal{H}^\delta(\Lambda \setminus \Lambda_{\mathrm{rad}}) = 0$ where $\delta := \dim_H(\Lambda_{\mathrm{rad}})$, then
   either $\dim_H(\Lambda) > \dim_T(\Lambda)$ or Λ is a real-analytic manifold.

Dimension spectra

Definition

The **(Hausdorff) dimension spectrum** of a CIFS $\mathcal{U} = (u_a)_{a \in E}$ is

$$\mathsf{DimSpec}(\mathcal{U}) := \{\dim_H(F_A) : A \subseteq E\}.$$

CLU proved that an infinite CIFS dimension spectrum is compact and perfect, and conjectured that every such set might be realized. Also asked if such a set could be uniformly perfect. We could prove

Theorem (DS, JFG 2022)

$\exists$ *a compact and perfect set* $K \subseteq [0, 1]$ *such that* $0 \in K$ *and* $\mathsf{DimSpec}(\mathcal{U}) \neq K$ *for all CIFSes* $\mathcal{U}$ *on* $\mathbb{R}$.

Theorem (DS, JFG 2022)

$\exists$ *an infinite SIFS on* $\mathbb{R}$ *whose dimension spectrum has Hausdorff dimension zero (and thus cannot be uniformly perfect).*

Conjectures and future work

Here is a conjecture on the "shape" of dimension spectra ...

> **Conjecture (DS, JFG 2022)**
>
> *The only sets $K \subseteq [0, \infty)$ such that both K and its mirror image $\sup(K) - K$ are dimension spectra of CIFSes are intervals, i.e. $K = [0, \lambda]$ for some $\lambda > 0$.*

Let $F \subseteq X$ metric space. The local Hausdorff dimension of F at $x \in X$ is

$$\dim_x(F) = \lim_{\varepsilon \to 0} \dim_H(F \cap B(x, \varepsilon))$$

> **Conjecture (DS, JFG 2022)**
>
> *Let $F \subseteq [0, \infty)$ be the dimension spectrum of a CIFS. The map $x \mapsto \dim_x(F)$ restricted to $x \in F$ is continuous and weakly decreasing.*

Classification of Dimension Spectra

Let $F \subseteq X$ metric space. The local Hausdorff dimension of F at $x \in X$ is

$$\dim_x(F) = \lim_{\varepsilon \to 0} \dim_H(F \cap B(x, \varepsilon))$$

Conjecture (DS, JFG 2022)

Let $F \subseteq [0, \infty)$ be the dimension spectrum of a CIFS. Then one of the following three mutually exclusive scenarios holds:

  I *F is equal to the union of finitely many intervals.*

  II *F has zero Hausdorff dimension.*

  III *The local Hausdorff dimension satisfies $\dim_x(F) = \min(1, c/x)$ for all $x \in F$, for some $0 < c < \sup(F)$, i.e. the graph of the function $F \ni x \mapsto \dim_x(F)$ is a horizontal line followed by a hyperbola.*

Cf. Sarnak's 2023 Chern Lectures (Bass Note Spectra)

Some Fractal Numerology

Jarník (1928)

$$0.25 < \dim_H(F_{\leq 2}) < 1$$

Good (1939)

$$0.5306 < \dim_H(F_{\leq 2}) < 0.5320$$

Bumby (1982)

$$0.5312 < \dim_H(F_{\leq 2}) < 0.5314$$

Hensley (1996)

$$0.53128049 < \dim_H(F_{\leq 2}) < 0.53128049$$

Falk–Nussbaum (2012-2018)

$$0.531280505981423 < \dim_H(F_{\leq 2}) < 0.531280506343388$$

Pollicott–Jenkinson (2001-2018)

$\dim_H(F_{\leq 2}) = 0.531280506277205141624468647368471785493059109018398779888397803927529535643831345918109570181185239 8 \ldots$

Wizards of Dimension Computation (1975-2018)

…with late 20th cen. PhDs (1960–1996)

Richard Bumby, Richard S. Falk, Philippe Flajolet, Doug Hensley, Oliver Jenkinson,
Leo P. Kadanoff, Curt McMullen, Roger Nussbaum, Mark Pollicott, Brigitte Vallée

Metamodern Wizardry (2011–)

…with early 21st cen. PhDs (2001–)

Polina Vytnova, Amit Priyadarshi, Julia Slipantschuk, Dmitriy Leykekhman,
Caroline Wormell, Vasileios Chousionis, Artur Avila, Sasha Skripchenko

New challenges for validated numerics

- ⋆ Compute $\dim_H(F_{\leq N})$ to $\boxed{2^{10} \text{ digits in } 2^2 \text{ hours on } 2^0 \text{ Laptop/PC}}$

- ⋆ Same $2^{(10,2,0)}$ problem for $\dim_H(F_{\text{primes}}) \sim 0.675$

- ⋆ Same $2^{(10,2,0)}$ problem for $\dim_H(F_{\text{even}}) \sim 0.719$

- ⋆ Same $2^{(10,2,0)}$ problem for $\dim_H(F_{\text{odd}}) \sim 0.8211$

- ⋆ Same $2^{(10,2,0)}$ problem for $\dim_H(F_{\geq N})$

Infinite alphabet

3 estimates above from Chousionis–Leykekhman–Urbański (TAMS, 2020)

Finite alphabet

Jenkinson–Pollicott (Advances, 2018) compute 100 digits of $\dim_H(F_{\leq 2})$ using periodic points and zetas
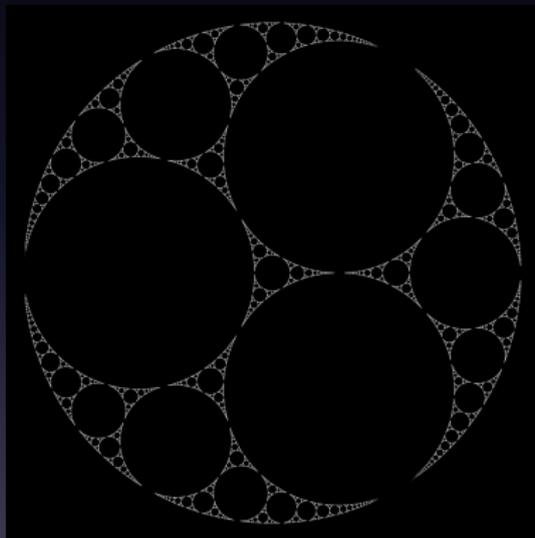
Bandtlow–Slipantschuk (arXiv, 2020) Lagrange approximation of transfer operators

Pollicott–Vytnova (arXiv, 2020) HD estimates for Lagrange & Markov spectra, Fuchsian limit sets

New challenges for validated numerics

First stated in June 2020 at Fractals And Related Fields (FARF) Online

2^{10} digits in 2^2 hours on 2^0 Laptop/PC

$1.300 < \dim_H(\Lambda) < 1.315$ (Boyd, 1973) $\dim_H(\Lambda) \sim 1.305688$ (McMullen, 1998)

(Vytnova–Wormwell, 2024) $\dim_H(\Lambda) = 1.30568672804987718464598620685104089110602644149646829644618838899698642050296986 \ldots$

Ideas/Questions/Comments/Suggestions/Solutions/Criticism Welcome!
🗁 https://www.uwlax.edu/profile/tdas/
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