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RESEARCH ARTICLE

# Hidden Symmetries of the Classical $3 \times 3$ Magic Square

**Vijender Nallapu**[1] **Mrinal Kanti Roychowdhury**[2]

[1]Department of Mathematics, National Institute of Technology Warangal, Warangal, India
[2]School of Mathematical and Statistical Sciences, The University of Texas Rio Grande Valley (UTRGV), Texas, USA

*Emails:* [1]vijendernallapu@nitw.ac.in *(corresponding author)*; [2]mrinal.roychowdhury@utrgv.edu

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**Abstract.** The classical $3 \times 3$ magic square is one of the most famous objects in recreational mathematics. Its defining feature is that every row, every column, and the two main diagonals have the same sum. However, beyond this well-known property, the square contains several hidden symmetries involving rotations, reflections, complementary pairs, parity, and balance around the central entry. In this article, we present an elementary exploration of these symmetries and show how the classical magic square provides a beautiful introduction to mathematical structure and pattern recognition.

**Keywords.** Magic Squares; Recreational Mathematics; Symmetry; Combinatorics; Number Patterns

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## 1. Introduction

A magic square is a square array of numbers arranged so that the sum of the entries in every row, every column, and the two main diagonals is the same. Magic squares have fascinated mathematicians, historians, and puzzle enthusiasts for centuries and occupy a prominent place in recreational mathematics [1, 2, 3, 4]. This common sum is called the *magic sum.*

The classical $3 \times 3$ magic square is

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

Each row, column, and diagonal has sum

$$15.$$

For example,

$$2+9+4=15, \qquad 7+5+3=15, \qquad 6+1+8=15,$$

and

$$2+5+8=15, \qquad 4+5+6=15.$$

Although the magic-sum property is the most visible feature of the square, it is not the only interesting one. The goal of this article is to explore some of the hidden symmetries of this familiar object.

The purpose of this article is not to develop a new theory of magic squares or to investigate advanced algebraic and number-theoretic phenomena associated with them. Rather, our goal is to present an accessible exploration of several elegant structural properties of the classical $3 \times 3$ magic square. We focus on elementary yet remarkable features such as the central role of the number 5, complementary pairs, parity balance, geometric symmetry, and structural uniqueness. These properties reveal how a simple arrangement of nine integers can exhibit a rich interplay between arithmetic, combinatorics, geometry, and elementary algebra.

The classical $3 \times 3$ magic square continues to inspire investigations from a variety of mathematical perspectives, ranging from recreational mathematics and mathematical education to combinatorics, algebra, number theory, and symmetry. The present article is intended as an elementary introduction to some of these structural ideas and as an invitation for readers to explore further developments and deeper connections.

## 2. The Classical $3 \times 3$ Magic Square

The classical square uses the integers

$$1, 2, 3, 4, 5, 6, 7, 8, 9$$

exactly once. Since their total sum is

$$1+2+\cdots+9=45,$$

and since the square has three rows, the common row sum must be

$$\frac{45}{3}=15.$$

Thus any $3 \times 3$ magic square using the numbers 1 through 9 must have magic sum 15.

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

Figure 1. The classical $3 \times 3$ magic square.

## 3. Historical Background: The Lo Shu Magic Square

The earliest known magic square is the Lo Shu square of ancient China. According to legend, a turtle emerged from the Luo River carrying a remarkable numerical pattern on its shell. This pattern was later interpreted as a $3 \times 3$ arrangement of the integers 1 through 9 possessing equal row, column, and diagonal sums. Historical accounts of the Lo Shu square and its influence on the development of mathematics may be found in [3, 5].

Although the historical details remain uncertain, the Lo Shu square has played an important role in Chinese mathematics, philosophy, astronomy, and art for nearly two thousand years.

Modern mathematics studies magic squares independently of these legends. Nevertheless, the historical story illustrates the enduring fascination that numerical symmetry has exerted across different cultures.

The classical magic square discussed throughout this paper is equivalent to the Lo Shu square up to rotation and reflection. For further historical and cultural perspectives on magic squares, see [3, 5].

## 4. The Central Role of the Number 5

The center of the square is occupied by the number 5. This is not accidental. The number 5 is the average of the numbers 1 through 9:

$$\frac{1 + 2 + \cdots + 9}{9} = 5.$$

The number 5 acts as the balancing point of the square. The entries opposite each other through the center form complementary pairs:

$$2 + 8 = 10, \qquad 9 + 1 = 10,$$

$$4 + 6 = 10, \qquad 7 + 3 = 10.$$

Since each such pair sums to 10, adding the center number gives

$$10 + 5 = 15.$$

This explains why the two diagonals and the middle row and column naturally have sum 15.

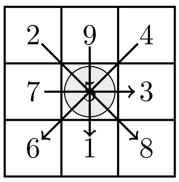


Figure 2. Opposite entries through the center form complementary pairs.

**Theorem 4.1.** *In every $3 \times 3$ magic square formed from the integers $1, 2, \ldots, 9$, the central entry must be 5.*

*Proof.* The sum of the integers from 1 to 9 is

$$1 + 2 + \cdots + 9 = 45.$$

Since there are three rows and each row has the same sum $M$, we obtain

$$3M = 45,$$

which implies

$$M = 15.$$

Let the center entry be $c$.
The four pairs of entries opposite each other through the center must each sum to

$$15 - c.$$

Since there are four such pairs, the sum of the eight non-central entries is

$$4(15 - c) = 60 - 4c.$$

On the other hand, the sum of the eight non-central entries is

$$45 - c.$$

Therefore

$$45 - c = 60 - 4c.$$

Hence

$$3c = 15,$$

and consequently

$$c = 5.$$

Therefore the center of every such magic square must be 5. □

## 5. Corner and Edge Symmetry

The four corner entries are

$$2, \ 4, \ 6, \ 8.$$

These are precisely the even numbers among $1, 2, \ldots, 9$. Their sum is

$$2 + 4 + 6 + 8 = 20.$$

The four edge-center entries are

$$1, \ 3, \ 7, \ 9.$$

These are the odd numbers except the central number 5. Their sum is also

$$1 + 3 + 7 + 9 = 20.$$

Thus the total contribution of the four corner entries equals the total contribution of the four edge-center entries.

$$\text{Corners sum} = \text{Edge-centers sum} = 20.$$

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

Figure 3. The corner entries and edge-center entries both sum to 20.

## 6. Uniqueness of the Classical Magic Square

One of the most remarkable facts about the $3 \times 3$ magic square is that, up to symmetry, there is essentially only one such square.

**Theorem 6.1.** *Every $3 \times 3$ magic square formed from the integers $1, 2, \ldots, 9$ is obtained from the classical magic square by a rotation or reflection.*

*Proof.* By Theorem 4.1, the center entry must be 5.

The remaining eight numbers may be grouped into the complementary pairs

$$(1, 9), \quad (2, 8), \quad (3, 7), \quad (4, 6),$$

Each of which sums to 10.

Since every line passing through the center must have magic sum 15, the entries opposite each other through the center must form one of these complementary pairs.

Write the square in the form

$$\begin{matrix} a & b & c \\ d & 5 & f \\ g & h & i \end{matrix}$$

Because opposite entries through the center are complementary,

$$i = 10 - a, \qquad h = 10 - b, \qquad g = 10 - c, \qquad f = 10 - d.$$

The top row and left column give

$$a + b + c = 15, \qquad a + d + g = 15.$$

Hence

$$b = 15 - a - c, \qquad d = 5 + c - a.$$

Thus, once the two upper corner entries $a$ and $c$ are chosen, every other entry is determined. In particular, $b = 15 - a - c$ and $d = 5 + c - a$ must both belong to $\{1, 2, \ldots, 9\}$, be distinct from $a, c, 5$, and their complementary entries $10 - b$ and $10 - d$ must also be distinct members of the same set. Checking the four complementary pairs $(1, 9)$, $(2, 8)$, $(3, 7)$, and $(4, 6)$ under these constraints leaves precisely the following possible ordered pairs for $(a, c)$:

$$(2, 4),\ (2, 6),\ (4, 2),\ (4, 8),\ (6, 2),\ (6, 8),\ (8, 4),\ (8, 6).$$

Substitution gives exactly the eight rotations and reflections of the classical square. Therefore no essentially different arrangement occurs: every valid square differs from the classical magic square only by a rotation or reflection.

Hence every $3 \times 3$ magic square is equivalent to the classical magic square under the symmetry group of the square. □

The uniqueness of the classical $3 \times 3$ magic square is one reason for its special place in recreational mathematics. Unlike higher-order magic squares, which occur in many different forms, the $3 \times 3$ case is completely determined by its symmetry.

## 7. Rotations and Reflections

A square possesses eight geometric symmetries: four rotations and four reflections. These symmetries form the dihedral group $D_4$. The relationship between magic squares and geometric symmetry is a classical topic in recreational mathematics [2, 6].

Applying any symmetry in $D_4$ to a magic square produces another magic square. Beginning with the classical $3 \times 3$ magic square, the four rotations generate three additional arrangements. These are not essentially different magic squares; rather, they represent the same square viewed from different orientations.

Original

| | | |
|---|---|---|
| 2 | 9 | 4 |
| 7 | 5 | 3 |
| 6 | 1 | 8 |

90°

| | | |
|---|---|---|
| 6 | 7 | 2 |
| 1 | 5 | 9 |
| 8 | 3 | 4 |

180°

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

270°

| | | |
|---|---|---|
| 4 | 3 | 8 |
| 9 | 5 | 1 |
| 2 | 7 | 6 |

Figure 4. Rotations of the classical magic square.

Similarly, the four reflections preserve the magic property.

Horizontal

| | | |
|---|---|---|
| 6 | 1 | 8 |
| 7 | 5 | 3 |
| 2 | 9 | 4 |

Vertical

| | | |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

Main diagonal

| | | |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

Other diagonal

| | | |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

Figure 5. Reflections of the classical magic square.

Each of the eight symmetric images remains a magic square with magic sum 15. Consequently, the magic square property is invariant under the action of the dihedral group $D_4$. Thus, the eight arrangements obtained by rotations and reflections constitute a single equivalence class under geometric symmetry.

## 8. The Dihedral Group $D_4$

The eight rotations and reflections of a square form a symmetry structure known in mathematics as the dihedral group $D_4$. For the purposes of this article, it suffices to observe that these symmetries preserve the magic property and generate all equivalent versions of the classical $3 \times 3$ magic square.

Thus, every rotation or reflection of the classical magic square produces another magic square with the same magic sum. This provides a simple illustration of how symmetry helps organize and explain mathematical structure.

## 9. Why the Magic Sum Is Preserved

The reason rotations and reflections preserve the magic property is simple: these transformations do not change the entries; they only change their positions.

Rows may become columns, columns may become rows, and diagonals may be interchanged. Since all rows, columns, and diagonals originally have sum 15, the transformed square also has the same property.

Thus the collection of eight symmetries preserves the structure of the magic square.

## 10. A Balance Around the Average

Another way to understand the square is to subtract 5 from every entry. This gives

| | | |
|---|---|---|
| −3 | 4 | −1 |
| 2 | 0 | −2 |
| 1 | −4 | 3 |

Now every row, column, and diagonal sums to 0.
For example,

$$-3 + 4 - 1 = 0,$$

$$2 + 0 - 2 = 0,$$

$$1 - 4 + 3 = 0.$$

This transformed square reveals that the original magic square is balanced around its average value 5.

| | | |
|---|---|---|
| -3 | 4 | -1 |
| 2 | 0 | -2 |
| 1 | -4 | 3 |

Figure 6. Subtracting 5 from every entry gives a zero-sum magic square.

## 11. The Complementary Pair Principle

The complementary pair principle is one of the simplest hidden rules of the classical magic square.

The numbers 1 through 9 may be paired as

$$(1, 9), \quad (2, 8), \quad (3, 7), \quad (4, 6),$$

with 5 left in the center. Each pair sums to 10.

Every line through the center consists of one complementary pair together with 5. Therefore each such line has sum

$$10 + 5 = 15.$$

This explains the two diagonals, the middle row, and the middle column.

The remaining rows and columns are then forced by the total balance of the square.

## 12. Exercises for the Reader

The following exercises provide further opportunities to explore the structure of the classical magic square.

Many additional puzzles and explorations involving magic squares may be found in the recreational mathematics literature [7, 8].

1. Verify directly that every row, column, and diagonal has sum 15.

2. Show that the sum of the four corner entries equals the sum of the four edge-center entries.

3. Verify that subtracting 5 from every entry produces a square whose rows, columns, and diagonals all sum to zero.

4. Construct all eight symmetric versions of the classical magic square.

5. Investigate which of the properties discussed in this article continue to hold for a $4 \times 4$ magic square.

## 13. Conclusion

The classical $3 \times 3$ magic square is much more than a simple arrangement of numbers. Beneath the visible magic-sum property lies a rich structure involving complementary pairs, parity balance, rotations, reflections, and symmetry around the central value. These features make the square an ideal object for recreational mathematics: it is elementary enough to be accessible to students while naturally illustrating invariance, structural balance, combinatorics, and the action of the dihedral group $D_4$.

Its enduring appeal comes from the remarkable interplay of arithmetic, geometry, and elementary algebra within only nine entries. Readers interested in further explorations of magic squares, numerical patterns, and related recreational topics are encouraged to consult [1, 2, 3, 4].

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## Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

## Data Availability Statement

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