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Research Article

Curious Patterns and Identities Involving Consecutive Integers

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Abstract. Consecutive integers exhibit numerous surprising patterns and elegant identities. In this article, we investigate several classical properties involving products, sums, divisibility, perfect squares, cubes, pronic numbers, triangular numbers, sums of squares, and Pythagorean triples. Using only elementary algebra, symmetry arguments, and mathematical induction, we establish a collection of identities that reveal rich connections among these classical topics. The exposition is intended for students, teachers, and recreational mathematicians, illustrating how simple observations about consecutive integers can lead to beautiful mathematical patterns and deeper insights into elementary number theory.

Keywords. Consecutive integers; divisibility; number patterns; recreational mathematics; elementary number theory.

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1. Introduction

Consecutive integers are among the simplest and most familiar objects in mathematics. They arise naturally in arithmetic, algebra, combinatorics, and number theory, and are often encountered at an early stage of mathematical education. Despite their apparent simplicity, consecutive integers exhibit a remarkable variety of patterns, identities, and divisibility properties.

Many classical results involving consecutive integers can be established using only elementary algebra. For example, the product of two consecutive integers is always even, the product of three consecutive integers is always divisible by six, and certain products of four consecutive integers differ from perfect squares by exactly one. Such observations have fascinated mathematicians for centuries and continue to provide attractive examples in recreational mathematics [1, 2].

The study of consecutive integers is closely connected with the theory of figurate numbers, including triangular numbers and square numbers, which have been investigated since antiquity.

These classical number patterns continue to play an important role in elementary number theory and recreational mathematics, illustrating how simple numerical relationships can lead to elegant mathematical discoveries. Many of the classical identities discussed throughout this paper may be found in standard texts on elementary number theory, discrete mathematics, and recreational mathematics; see, for example, [3, 4, 5, 6].

The study of consecutive integers also offers an excellent opportunity to develop mathematical reasoning. Simple numerical experiments often suggest unexpected patterns, which may then be confirmed through rigorous proofs. In this way, consecutive integers serve as a natural bridge between elementary computation and abstract mathematical thinking.

The purpose of this article is to present a collection of elegant results concerning consecutive integers. We examine products and sums of consecutive integers, divisibility properties, perfect squares, cubes, pronic numbers, triangular numbers, sums of squares, and classical identities involving Pythagorean triples, emphasizing elementary proofs and mathematical insight. The exposition is intended for students, teachers, and recreational mathematicians, illustrating how surprisingly rich mathematics can emerge from some of the simplest sequences of numbers and how these elementary ideas connect naturally with classical topics in number theory.

Throughout the paper, n denotes an integer unless otherwise specified.

2. Products of Consecutive Integers

We begin with a fundamental observation.

Proposition 2.1. *For every integer n, the product $n(n+1)$ is even.*

Proof. Among any two consecutive integers, exactly one is even. Therefore their product is even. □

The next result strengthens the preceding proposition.

Proposition 2.2. *For every integer n, the product*

$$n(n+1)(n+2)$$

is divisible by 6.

Proof. Among any three consecutive integers, exactly one is divisible by 3, since multiples of 3 occur every third integer. Furthermore, at least one of the three integers is even. Hence the product contains the factors 2 and 3, and therefore is divisible by

$$2 \cdot 3 = 6.$$

□

The preceding proposition illustrates the strong divisibility properties possessed by products of consecutive integers. Surprisingly, products of four consecutive integers satisfy an even more remarkable property: after adding one, the result is always a perfect square.

Theorem 2.3. *For every integer n,*

$$n(n+1)(n+2)(n+3)+1$$

is a perfect square.

Proof. Observe that

$$n(n+3) = n^2 + 3n$$

and

$$(n+1)(n+2) = n^2 + 3n + 2.$$

Let

$$x = n^2 + 3n.$$

Then

$$x(x+2) + 1 = x^2 + 2x + 1 = (x+1)^2 = (n^2 + 3n + 1)^2.$$

Thus

$$n(n+1)(n+2)(n+3) + 1 = (n^2 + 3n + 1)^2,$$

which is a perfect square. □

The first few examples are

$$1 \cdot 2 \cdot 3 \cdot 4 + 1 = 5^2,$$

$$2 \cdot 3 \cdot 4 \cdot 5 + 1 = 11^2,$$

and

$$3 \cdot 4 \cdot 5 \cdot 6 + 1 = 19^2.$$

These examples illustrate the theorem and reveal a surprisingly regular pattern generated by products of four consecutive integers.

3. Sums of Consecutive Integers

Products of consecutive integers exhibit remarkable divisibility properties. Equally interesting patterns arise when we consider sums of consecutive integers. Many classical formulas for such sums can be established using only elementary algebra and reveal elegant numerical relationships.

We begin with the well-known formula for the sum of the first n positive integers.

Theorem 3.1 (Gauss' Formula)**.** *For every positive integer n,*

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Proof. Write the sum both forwards and backwards:

$$S = 1 + 2 + \cdots + (n-1) + n,$$

and

$$S = n + (n-1) + \cdots + 2 + 1.$$

Adding these two equations term by term gives

$$2S = (n+1) + (n+1) + \cdots + (n+1) = n(n+1).$$

Hence

$$S = \frac{n(n+1)}{2}.$$

□

This elegant identity is traditionally attributed to Carl Friedrich Gauss and is one of the best-known formulas in elementary mathematics. It appears in many standard texts on combinatorial reasoning and elementary number theory; see, for example, [3, 6].

The preceding theorem immediately leads to a formula for the sum of any collection of consecutive integers.

Proposition 3.2. *For integers n and $k \geq 0$,*

$$n + (n+1) + \cdots + (n+k) = \frac{(k+1)(2n+k)}{2}.$$

Proof. There are $k+1$ terms. The first term is n and the last term is $n+k$. Therefore the average of the first and last terms is

$$\frac{n + (n+k)}{2} = \frac{2n+k}{2}.$$

Multiplying this average by the number of terms gives

$$\frac{(k+1)(2n+k)}{2},$$

which is the required sum. □

We now investigate a particularly elegant special case involving an odd number of consecutive integers.

Proposition 3.3. *The sum of three consecutive integers is divisible by 3.*

Proof. Let the integers be

$$n-1, \quad n, \quad n+1.$$

Then

$$(n-1) + n + (n+1) = 3n,$$

which is divisible by 3. □

A similar phenomenon occurs for five consecutive integers.

Proposition 3.4. *The sum of five consecutive integers is divisible by 5.*

Proof. Let the integers be

$$n-2, \quad n-1, \quad n, \quad n+1, \quad n+2.$$

Then

$$(n-2) + (n-1) + n + (n+1) + (n+2) = 5n,$$

which is divisible by 5. □

The preceding examples suggest a more general result.

Theorem 3.5. *For every positive integer k, the sum of $2k+1$ consecutive integers is divisible by $2k+1$.*

Proof. Consider the integers

$$n-k,\ n-k+1,\ \ldots,\ n+k.$$

Pairing terms symmetrically about the middle integer n, we obtain

$$(n-k)+(n+k)=2n,$$

$$(n-k+1)+(n+k-1)=2n,$$

and similarly for the remaining pairs.

Since there are k such pairs, their total contribution is

$$2kn.$$

Adding the middle term gives

$$2kn+n=(2k+1)n.$$

Hence the sum equals

$$(2k+1)n,$$

which is divisible by $2k+1$. □

The preceding theorem can also be obtained directly from the general formula by taking the first term to be $n-k$ and the last term to be $n+k$. This illustrates how the classical formula for consecutive sums naturally leads to the divisibility property for an odd number of consecutive integers.

The theorem has an appealing interpretation.

Corollary 3.6. *The average of an odd number of consecutive integers is the middle integer.*

Proof. From the theorem, the sum of

$$n-k,\ n-k+1,\ \ldots,\ n+k$$

is

$$(2k+1)n.$$

Dividing by the number of terms, namely $2k+1$, gives

$$n.$$

Therefore the average equals the middle integer. □

For example,

$$7+8+9+10+11=45, \qquad \frac{45}{5}=9,$$

which is precisely the middle integer.

Likewise,

$$17+18+19+20+21+22+23=140, \qquad \frac{140}{7}=20,$$

again giving the middle integer.

These examples illustrate the elegant symmetry inherent in sums of consecutive integers.

4. Consecutive Squares

The sequence of perfect squares

$$1, 4, 9, 16, 25, \ldots$$

has fascinated mathematicians for centuries. Consecutive squares exhibit several beautiful patterns that can be established using elementary algebra.

Proposition 4.1. *For every integer n,*

$$(n+1)^2 - n^2 = 2n + 1.$$

Consequently, the difference between consecutive squares is always an odd integer.

Proof. Expanding $(n+1)^2$ gives

$$(n+1)^2 = n^2 + 2n + 1.$$

Subtracting n^2 from both sides yields

$$(n+1)^2 - n^2 = 2n + 1.$$

Since $2n + 1$ is odd, the difference between consecutive squares is always an odd integer. □

The identity also admits an interesting interpretation. Since

$$2n + 1 = n + (n + 1),$$

we obtain

$$(n+1)^2 - n^2 = n + (n + 1).$$

Thus, the difference between two consecutive squares is equal to the sum of the corresponding consecutive integers. This elegant relationship provides another beautiful connection between consecutive integers and perfect squares.

The first few examples are

$$2^2 - 1^2 = 3 = 1 + 2,$$

$$3^2 - 2^2 = 5 = 2 + 3,$$

$$4^2 - 3^2 = 7 = 3 + 4,$$

and

$$5^2 - 4^2 = 9 = 4 + 5.$$

These examples illustrate that the odd integers arise naturally both as differences of consecutive squares and as sums of consecutive integers.

The preceding proposition leads to a classical identity.

Theorem 4.2. *For every positive integer n,*

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

Proof. We proceed by induction on n.

For $n = 1$,

$$1 = 1^2,$$

so the result holds.

Assume that

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

Then

$$1 + 3 + 5 + \cdots + (2n - 1) + (2n + 1) = n^2 + (2n + 1) = n^2 + 2n + 1.$$

Since

$$n^2 + 2n + 1 = (n + 1)^2,$$

we obtain

$$1 + 3 + 5 + \cdots + (2n + 1) = (n + 1)^2.$$

Therefore the result holds for $n + 1$. By mathematical induction, the theorem is true for all positive integers n. □

The theorem shows that every perfect square can be expressed as a sum of consecutive odd integers. For example,

$$25 = 1 + 3 + 5 + 7 + 9,$$

and

$$49 = 1 + 3 + 5 + 7 + 9 + 11 + 13.$$

We conclude with another simple observation.

Proposition 4.3. *The square of every odd integer is odd.*

Proof. Let

$$m = 2k + 1$$

be an odd integer.

Then

$$m^2 = (2k + 1)^2.$$

Expanding gives

$$4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$$

Hence m^2 is of the form $2r + 1$ for some integer r, and therefore is odd. □

The study of consecutive squares reveals a remarkable connection between algebraic identities and numerical patterns. In the next section we investigate analogous properties involving consecutive cubes.

5. Consecutive Cubes

The study of consecutive cubes reveals several elegant identities that complement those obtained for consecutive squares. Although the algebra becomes slightly more involved, many beautiful patterns continue to emerge.

Proposition 5.1. *For every integer n,*

$$(n+1)^3 - n^3 = 3n^2 + 3n + 1.$$

Proof. Expanding $(n+1)^3$ yields $(n+1)^3 = n^3 + 3n^2 + 3n + 1$.

Subtracting n^3 from both sides gives $(n+1)^3 - n^3 = 3n^2 + 3n + 1$.

This completes the proof. □

The first few differences are

$$2^3 - 1^3 = 7,$$
$$3^3 - 2^3 = 19,$$
$$4^3 - 3^3 = 37,$$

and

$$5^3 - 4^3 = 61.$$

Unlike consecutive squares, whose differences form an arithmetic progression, the differences between consecutive cubes increase quadratically.

We now present one of the most celebrated identities involving cubes.

Theorem 5.2. *For every positive integer n,*

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Proof. We proceed by induction on n.

For $n = 1$,

$$1^3 = \left(\frac{1 \cdot 2}{2}\right)^2,$$

so the statement holds.

Assume that

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Then

$$1^3 + 2^3 + \cdots + n^3 + (n+1)^3 = \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3.$$

Using the induction hypothesis, we obtain

$$\begin{aligned}\left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3 &= (n+1)^2\left(\frac{n^2}{4} + n + 1\right)\\ &= (n+1)^2\left(\frac{(n+2)^2}{4}\right)\\ &= \left(\frac{(n+1)(n+2)}{2}\right)^2.\end{aligned}$$

Therefore

$$1^3 + 2^3 + \cdots + (n+1)^3 = \left(\frac{(n+1)(n+2)}{2}\right)^2.$$

Hence the formula holds for $n + 1$, completing the induction. □

For example,

$$1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2,$$

and

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 = 15^2.$$

Thus the sum of the first n cubes is always a perfect square.

We conclude with a useful divisibility property.

Proposition 5.3. *For every integer n,*

$$n^3 - n$$

is divisible by 6.

Proof. Observe that

$$n^3 - n = n(n^2 - 1) = n(n-1)(n+1).$$

The integers

$$n-1, \quad n, \quad n+1$$

are three consecutive integers.

Among any three consecutive integers, one is divisible by 3 and at least one is even. Therefore their product is divisible by 6. Hence

$$6 \mid (n^3 - n).$$

This completes the proof. □

The identities in this section illustrate that simple sequences of consecutive integers often conceal surprisingly rich algebraic structures. In the next section we investigate further divisibility properties associated with consecutive integers.

6. Curious Divisibility Results

One of the most remarkable features of consecutive integers is the strong divisibility properties exhibited by their products. In this section we present several classical results that illustrate this phenomenon.

Proposition 6.1. *For every integer n,*

$$n(n+1)(n+2)(n+3)$$

is divisible by 24.

Proof. Among four consecutive integers, one is divisible by 4 and at least one other is divisible by 2. Hence their product contains a factor of

$$4 \cdot 2 = 8.$$

Furthermore, among four consecutive integers, one must be divisible by 3. Therefore the product is divisible by

$$8 \cdot 3 = 24.$$

Thus

$$24 \mid n(n+1)(n+2)(n+3).$$

□

The preceding proposition suggests a more general phenomenon.

Theorem 6.2. *Let k and n be positive integers. Then the product of k consecutive integers*

$$n(n+1)(n+2)\cdots(n+k-1)$$

is divisible by

$$k!.$$

Proof. Consider the binomial coefficient

$$\binom{n+k-1}{k} = \frac{n(n+1)(n+2)\cdots(n+k-1)}{k!}.$$

Therefore

$$\frac{n(n+1)(n+2)\cdots(n+k-1)}{k!}$$

is an integer, which implies that

$$k! \mid n(n+1)(n+2)\cdots(n+k-1).$$

This completes the proof. □

The next result is a classical divisibility theorem in elementary number theory.

Proposition 6.3. *For every integer n,*

$$n^5 - n$$

is divisible by 30*.*

Proof. Observe that

$$n^5 - n = n(n^4-1) = n(n^2-1)(n^2+1) = n(n-1)(n+1)(n^2+1).$$

The integers

$$n-1, \quad n, \quad n+1$$

are three consecutive integers. Hence one of them is divisible by 3, and at least one of them is divisible by 2. Therefore

$$6 \mid n(n-1)(n+1).$$

It remains to establish divisibility by 5.

If $5 \mid n$, then clearly

$$5 \mid (n^5 - n).$$

Suppose now that $5 \nmid n$. Then

$$n \equiv \pm1,\ \pm2 \pmod 5.$$

In either case,

$$n^4 \equiv 1 \pmod 5.$$

Consequently,

$$n^5 - n = n(n^4-1) \equiv 0 \pmod 5,$$

and hence

$$5 \mid (n^5 - n).$$

Since 5 and 6 are relatively prime, it follows that

$$30 \mid (n^5 - n).$$

This completes the proof. □

We conclude with a simple but useful observation.

Proposition 6.4. *For every integer n,*

$$n^2 + n$$

is even.

Proof. Observe that

$$n^2 + n = n(n + 1).$$

Since n and $n + 1$ are consecutive integers, one of them is even. Therefore their product is even. □

The results of this section demonstrate that consecutive integers possess unexpectedly strong divisibility properties. In the next section we examine several additional numerical patterns generated by consecutive integers.

7. Interesting Number Patterns

In the previous sections we encountered several algebraic identities and divisibility properties involving consecutive integers. We now examine a few additional numerical patterns that arise naturally from consecutive integers. These patterns connect consecutive integers with figurate numbers, perfect squares, and elegant algebraic identities that have fascinated mathematicians for centuries.

7.1. Pronic Numbers

The numbers

$$n(n + 1)$$

are known as *pronic numbers*. They are obtained by multiplying two consecutive integers. The first few pronic numbers are

$$0,\ 2,\ 6,\ 12,\ 20,\ 30,\ 42, \ldots$$

Pronic numbers appear naturally in combinatorics, geometry, and elementary number theory; see, for example, [7, 8]. For example,

$$n(n + 1) = 2\binom{n+1}{2},$$

showing that every pronic number is twice a triangular number.

Proposition 7.1. *Every pronic number is even.*

Proof. A pronic number has the form

$$n(n + 1).$$

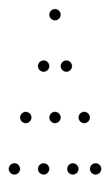
Since one of the consecutive integers n and $n + 1$ is even, their product is even. □

7.2. Triangular Numbers

The *nth triangular number* is defined by

$$T_n = 1 + 2 + \cdots + n.$$

The name *triangular number* comes from arranging equally spaced dots into a triangular pattern. For example,



contains

$$1 + 2 + 3 + 4 = 10$$

dots, illustrating that

$$T_4 = 10.$$

Using Gauss' formula,

$$T_n = \frac{n(n+1)}{2}.$$

The first few triangular numbers are

$$1,\ 3,\ 6,\ 10,\ 15,\ 21, \ldots$$

Notice that the numerator in the formula for T_n is precisely the corresponding pronic number.

Proposition 7.2. *For every positive integer n,*

$$8T_n + 1 = (2n+1)^2.$$

Consequently, eight times every triangular number plus one is a perfect square.

Proof. Using the formula for triangular numbers,

$$8T_n + 1 = 8\left(\frac{n(n+1)}{2}\right) + 1 = 4n(n+1) + 1.$$

Since

$$4n(n+1) + 1 = 4n^2 + 4n + 1 = (2n+1)^2,$$

the result follows. □

Another elegant identity connects triangular numbers with perfect squares.

Proposition 7.3. *For every positive integer n,*

$$T_n + T_{n-1} = n^2.$$

Proof. Using the formula for triangular numbers,

$$T_n + T_{n-1} = \frac{n(n+1)}{2} + \frac{n(n-1)}{2}.$$

Hence

$$T_n + T_{n-1} = \frac{n(n+1) + n(n-1)}{2} = \frac{2n^2}{2} = n^2.$$

This proves the identity. □

For example,

$$1 + 3 = 4, \qquad 3 + 6 = 9, \qquad 6 + 10 = 16, \qquad 10 + 15 = 25.$$

Thus the sum of two consecutive triangular numbers is always a perfect square.

7.3. Difference of Squares

Consecutive integers also give rise to the classical identity

$$(n+1)^2 - (n-1)^2 = 4n.$$

Proposition 7.4. *For every integer n,*

$$(n+1)^2 - (n-1)^2 = 4n.$$

Proof. Expanding both squares gives

$$(n+1)^2 = n^2 + 2n + 1$$

and

$$(n-1)^2 = n^2 - 2n + 1.$$

Subtracting the two equations yields

$$(n+1)^2 - (n-1)^2 = 4n.$$

This completes the proof. □

7.4. Difference of Consecutive Cubes

The difference between consecutive cubes also admits a simple and elegant algebraic expression.

Proposition 7.5. *For every integer n,*

$$(n+1)^3 - n^3 = 3n(n+1) + 1.$$

Proof. Expanding $(n+1)^3$ gives

$$(n+1)^3 = n^3 + 3n^2 + 3n + 1.$$

Subtracting n^3 yields

$$(n+1)^3 - n^3 = 3n^2 + 3n + 1 = 3n(n+1) + 1.$$

This completes the proof. □

Notice that the difference between consecutive cubes differs from three times the corresponding pronic number by exactly one.

7.5. Sums of Squares and Pythagorean Triples

Perfect squares also arise naturally through sums of squares. One of the oldest and most celebrated examples is provided by *Pythagorean triples*, namely positive integers (a, b, c) satisfying

$$a^2 + b^2 = c^2.$$

Classical treatments of Pythagorean triples may be found in [5, 8, 9]. The smallest and most familiar example is

$$3^2 + 4^2 = 5^2.$$

Other classical examples include

$$5^2 + 12^2 = 13^2,$$

$$8^2 + 15^2 = 17^2,$$

and

$$7^2 + 24^2 = 25^2.$$

Interestingly, the smallest Pythagorean triple consists of the three consecutive integers 3, 4, and 5. Although most Pythagorean triples do not consist entirely of consecutive integers, they provide another beautiful connection between integers and perfect squares.

The sum of the first n squares is also given by the classical identity

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6},$$

which plays a fundamental role in number theory, combinatorics, and discrete mathematics. This classical summation formula appears in many standard texts on elementary mathematics; see, for example, [6].

Notice that the numerator contains the pronic number $n(n + 1)$, once again illustrating how products of consecutive integers naturally appear in classical formulas.

For example,

$$1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30 = \frac{4 \cdot 5 \cdot 9}{6}.$$

These identities provide yet another illustration of how consecutive integers naturally generate elegant numerical patterns involving perfect squares.

The examples presented in this section demonstrate how consecutive integers naturally give rise to figurate numbers, perfect squares, sums of squares, and polynomial identities. Such elegant relationships have fascinated mathematicians for centuries and continue to provide beautiful illustrations of the richness of elementary number theory and recreational mathematics.

Taken together, the identities presented throughout this paper illustrate how elementary algebra and number theory reveal surprisingly rich mathematical structures hidden within the simple sequence of consecutive integers. We conclude by summarizing the principal themes and significance of these results.

8. Conclusion

Consecutive integers are among the simplest objects in mathematics, yet they give rise to a remarkable variety of elegant patterns, identities, and divisibility properties. In this article we explored numerous relationships involving products, sums, perfect squares, cubes, figurate numbers, and classical number-theoretic identities generated by consecutive integers. Although the individual results are elementary, together they reveal a rich mathematical structure that connects several important areas of mathematics.

We showed that products of consecutive integers possess strong divisibility properties, that differences of consecutive squares generate the odd integers, and that products of four consecutive integers satisfy a remarkable perfect-square identity. We also investigated classical formulas for sums of consecutive integers, the arithmetic properties of pronic and triangular numbers, identities involving sums of squares and cubes, and the appearance of consecutive integers in well-known results such as Pythagorean triples. These examples demonstrate how seemingly simple numerical patterns are closely related through elegant algebraic identities.

One of the most appealing aspects of these results is that they require only a modest mathematical background while illustrating a wide range of mathematical ideas, including algebraic manipulation, divisibility arguments, symmetry, mathematical induction, pattern recognition, and generalization. Consequently, they provide excellent examples for classroom instruction, mathematical enrichment, and recreational mathematics, serving as a natural bridge between elementary computation and rigorous mathematical proof.

The identities presented in this paper illustrate a recurring theme in mathematics: simple numerical patterns often conceal surprisingly rich algebraic structure. Many of the results discussed here admit interesting extensions involving arithmetic progressions, figurate numbers, binomial coefficients, polynomial identities, and other topics in elementary number theory. Exploring such generalizations offers valuable opportunities for students to develop mathematical intuition while appreciating the beauty and interconnectedness of mathematics.

The study of consecutive integers continues to provide a fertile source of elegant problems, unexpected discoveries, and mathematical insight. We hope that the examples presented here encourage readers not only to appreciate these classical identities but also to investigate new patterns of their own, reinforcing the enduring idea that profound mathematics can emerge from the simplest of numerical observations. Many additional identities, historical remarks, and recreational problems related to consecutive integers and elementary number theory may be found in the literature on recreational mathematics and number theory; see, for example, [1, 2, 4, 10, 11].

Declarations

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